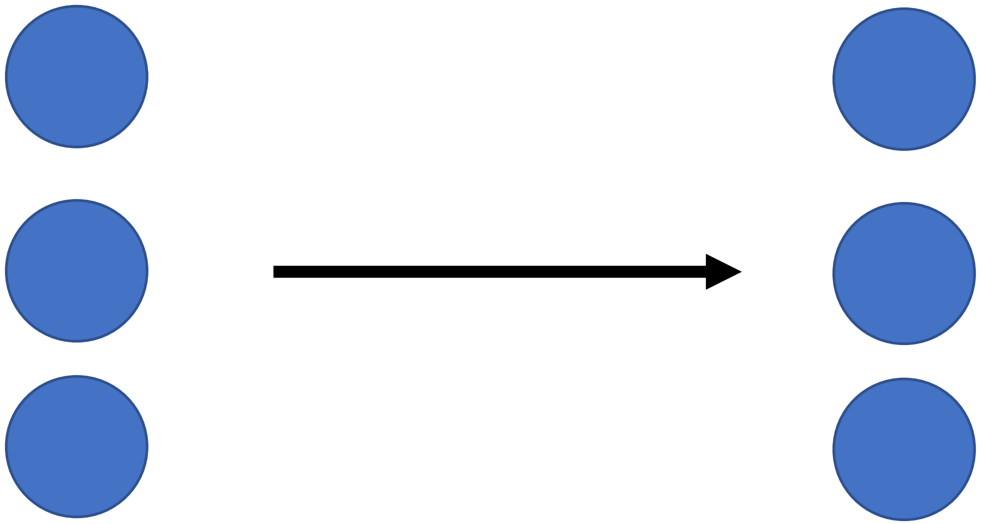
Domain-Indexing Variational Bayes: Interpretable Domain Index for Domain Adaptation

Zihao Xu*, Guang-Yuan Hao*, Hao He, Hao Wang

*These authors contributed equally to this work.

Domain Adaptation



Multiple Source Domains

X_s and Y_s

Multiple Target Domains

X_t predict Y_t

Using domain indices boosts Domain Adaptation performance [1][2].

[1] Hao Wang, Hao He, and Dina Katabi. Continuously indexed domain adaptation. In ICML, 2020.
[2] Zihao Xu, Guang-He Lee, Yuyang Wang, Hao Wang, et al. Graph-relational domain adaptation. In ICLR, 2022.

What is a domain index?

Domain index

- A real-value scalar (vector)
- Uniquely identify a domain
- Represent domain semantics

Background

Example for domain indices

Example: Gender Classification

Use average age as domain indices

Domain 1 19 Domain 2 25

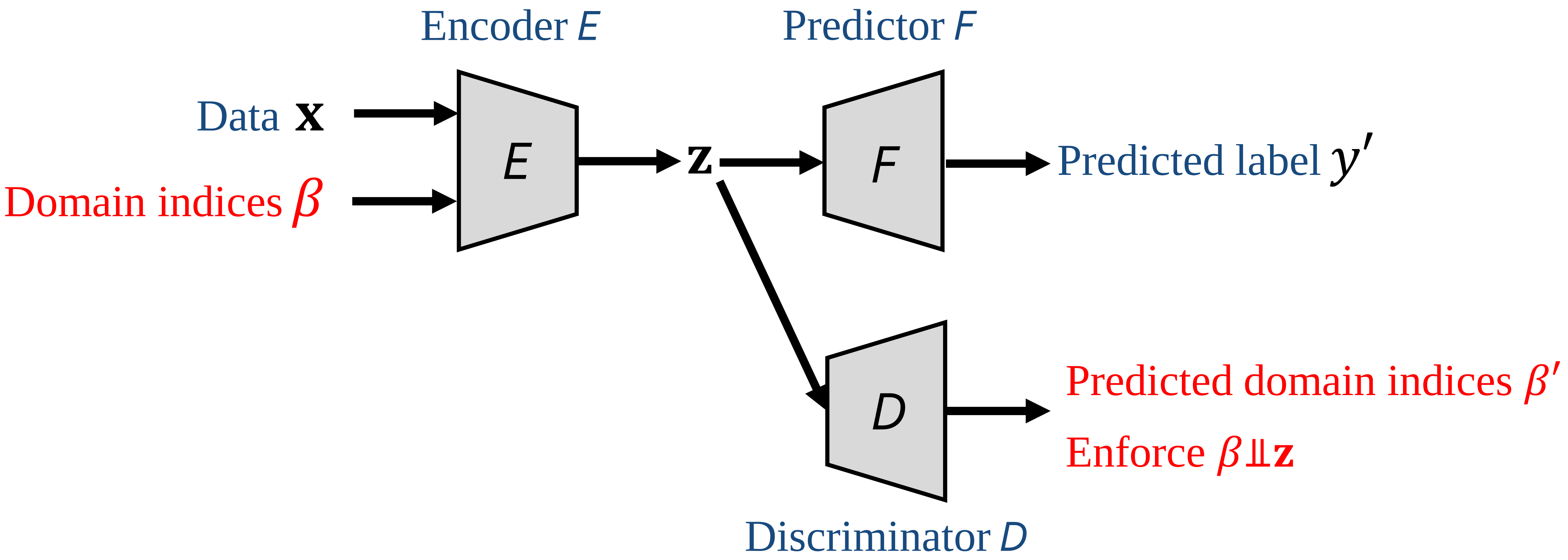
Domain 3 74 Domain 4 80

Source Domains: Young

Target Domains: Old

Exploiting effective facial patches for robust gender recognition. Tsinghua Science and Technology. 2019

Continuously indexed domain adaptation



Continuously indexed domain adaptation, ICML 2020

Challenge for using domain indices

Domain Indices may not be available!

Can we infer the domain indices from data?

Yes!

Advantages of inferring domain indices

- Improve interpretability of domain adaptation
- Improve performance of domain adaptation

How to infer the domain indices?

Our solution:

1. Rigorously define domain indices.

2. Based on our definition, use Probabilistic Graphical Model (PGM) to infer the domain indices.

How to infer the domain indices?

Our solution:

1. Rigorously define domain indices.

2. Based on our definition, use Probabilistic Graphical Model (PGM) to infer the domain indices.

Domain index definition (informal)

- Independence: independent of the data's encoding z
- Information Preservation: retain as much information on the data x as possible.
- Label Sensitivity: maximize adaptation performance.

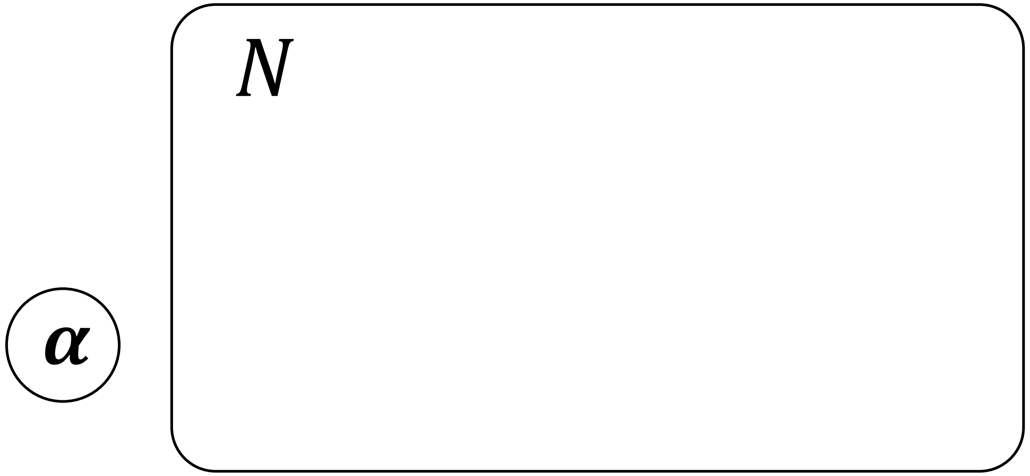
How to infer the domain indices?

Our solution:

1. Rigorously define domain indices.

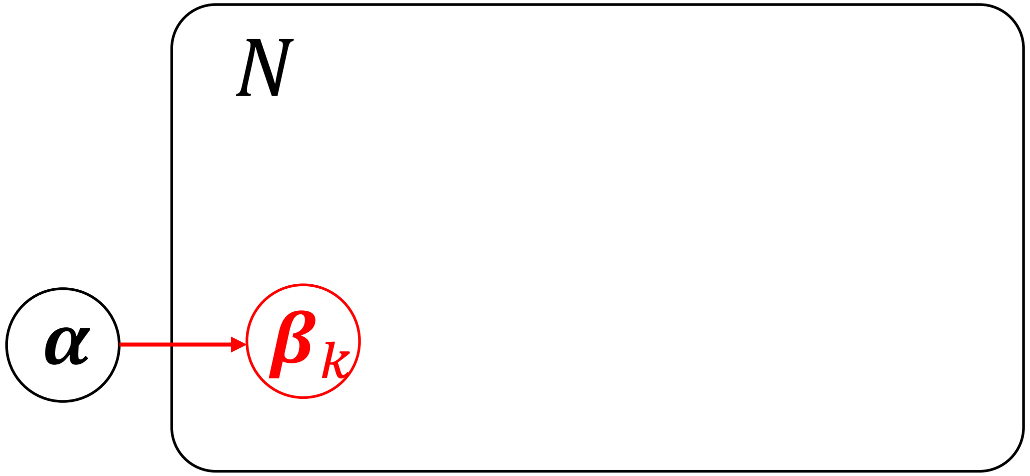
2. Based on our definition, use Probabilistic Graphical Model (PGM) to infer the domain indices.

PGM for inferring domain indices



α: prior of the domain index, k: domain ID

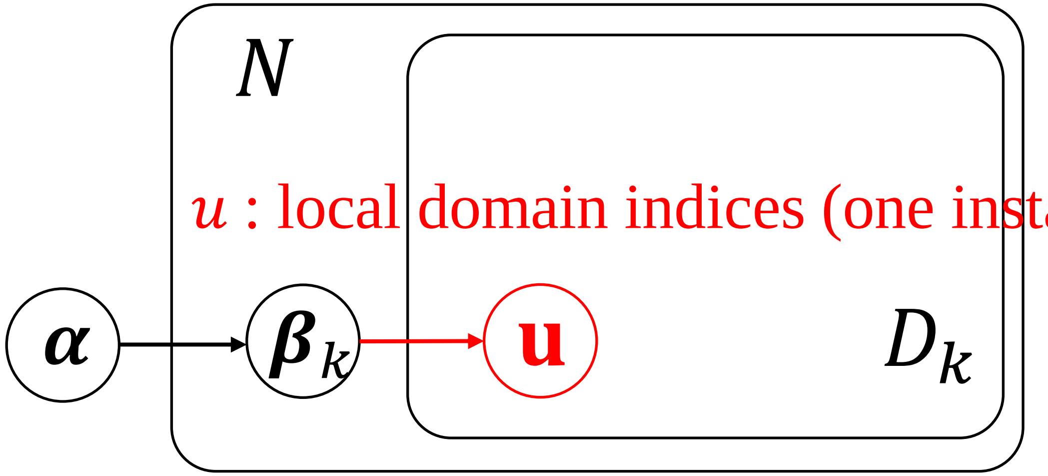
PGM for inferring domain indices



β_k: global domain indices (one instance per domain)

α: prior of the domain index, k: domain ID

PGM for inferring domain indices

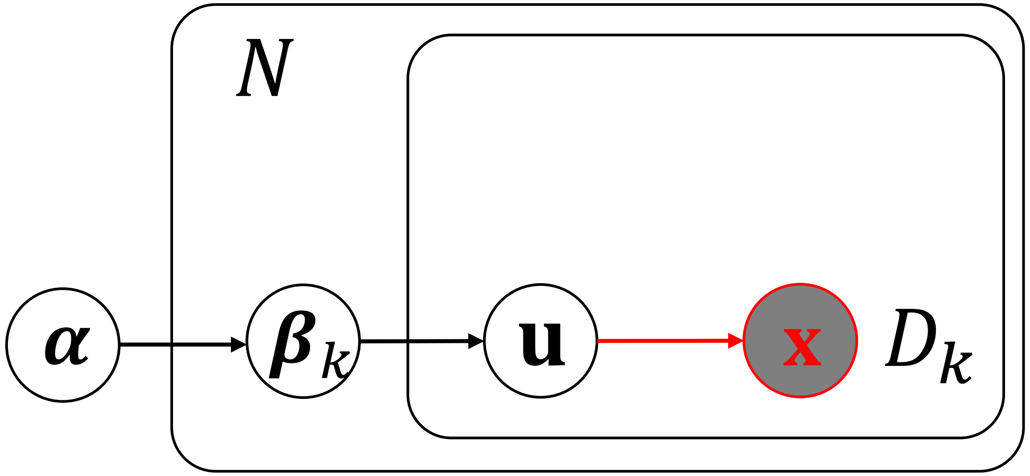


u : local domain indices (one instance per datum)

β_k: global domain indices (one instance per domain)

α: prior of the domain index, k: domain ID

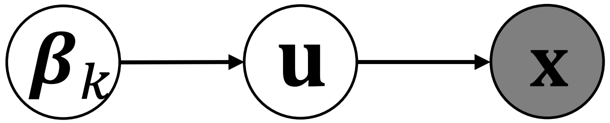
PGM for inferring domain indices



x: observed data (e.g., an image)

α: prior of the domain index, k: domain ID,
β_k: global domain indices (one instance per domain),
u: local domain indices (one instance per datum)

Intuition behind global/local domain indices



Average Age: 25.0

Age: 24.7

Age: 25.4

Age: 25.1

Age: 24.8

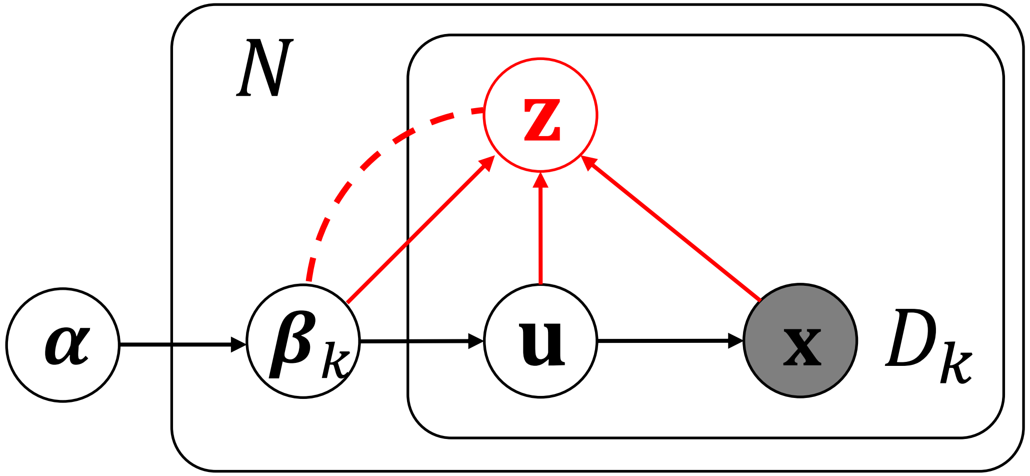
Global Domain Indices
(One per domain)

Local Domain Indices
(One per datum)

Observed data

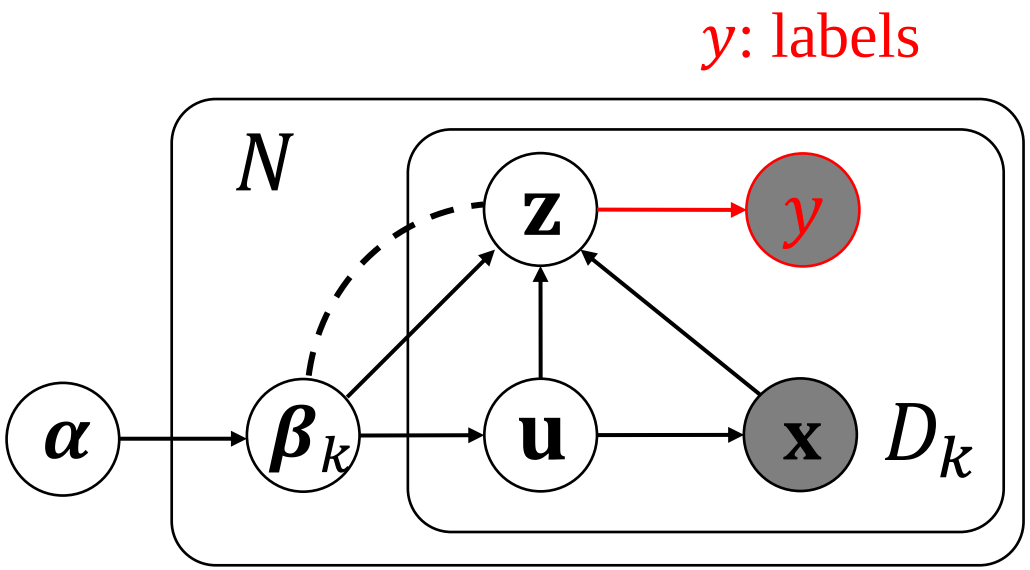
PGM for inferring domain indices

z: domain-invariant data embeddings, $\mathbf{z} \perp\!\!\!\perp \beta$



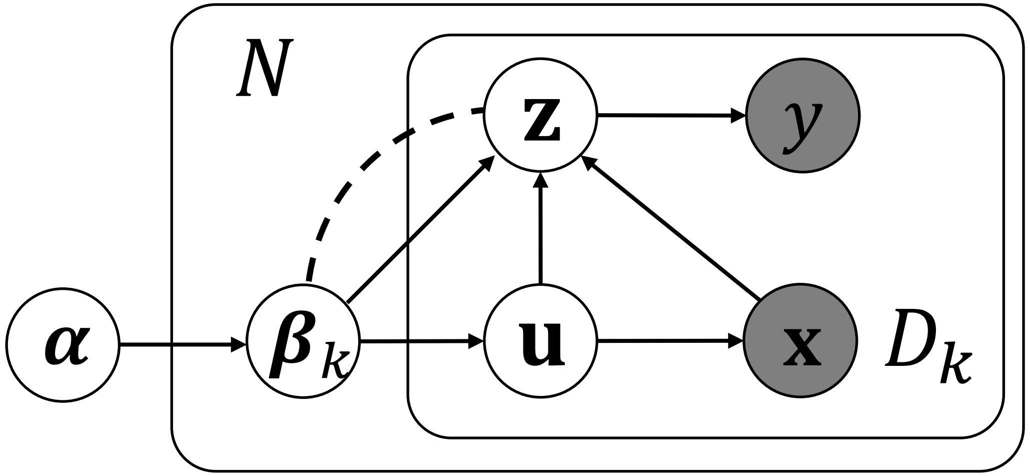
α: prior of the domain index, k: domain ID,
β_k: global domain indices (one instance per domain),
u: local domain indices (one instance per datum),
x: observed data

PGM for inferring domain indices

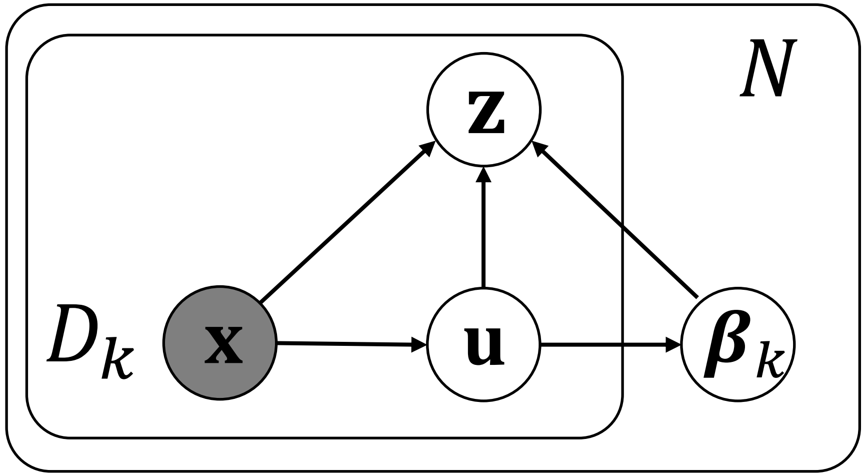


α: prior of domain index, k: domain ID,
β_k: global domain indices (one instance per domain),
u: local domain indices (one instance per datum),
x: observed data, z: domain-invariant data embeddings

Inference model



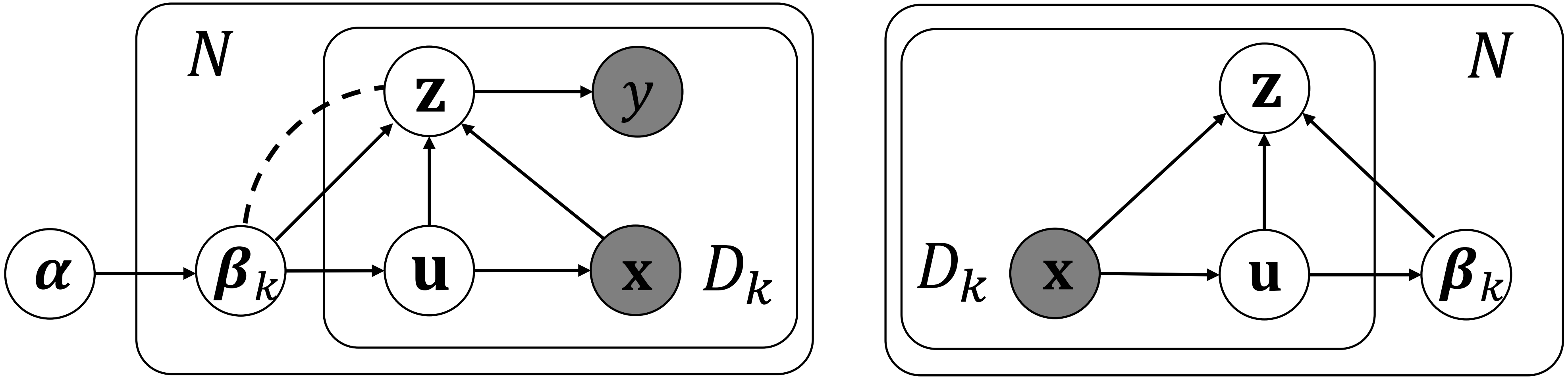
Generative model



Inference model

α: prior of domain index, k: domain ID,
β_k: global domain indices (one instance per domain),
u: local domain indices (one instance per datum),
x: observed data, z: domain-invariant data embeddings, y: labels

Variational domain-indexing (VDI)



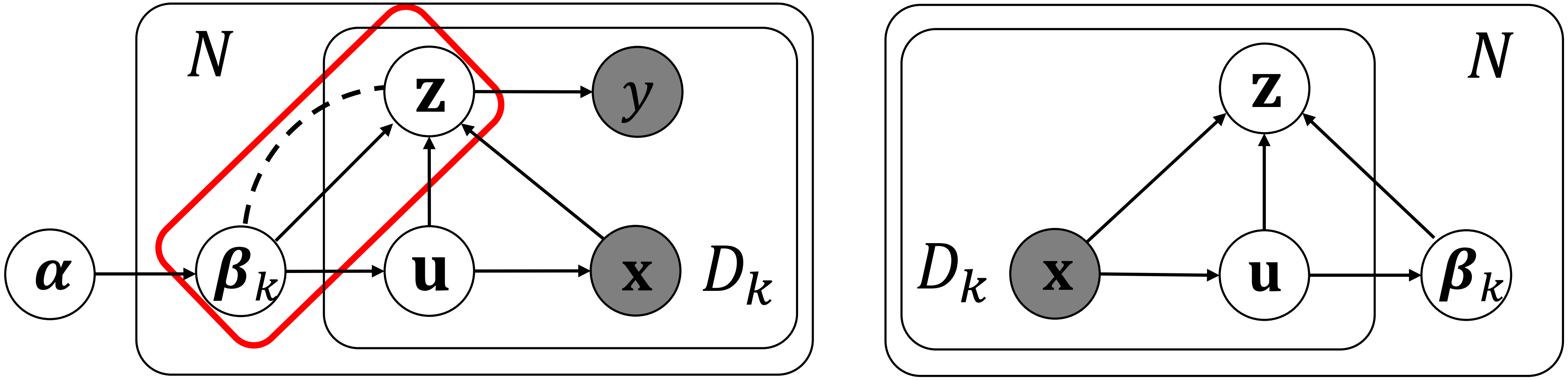
Maximize Evidence Lower Bound (ELBO):

$$\mathcal{L}_{ELBO}(\mathbf{x}, y) = \mathbb{E}_{q_\phi(\mathbf{u},\boldsymbol{\beta},\mathbf{z}|\mathbf{x})}[p_\theta(\mathbf{x}, \mathbf{u}, \boldsymbol{\beta}, \mathbf{z}, y|\boldsymbol{\alpha})] - \mathbb{E}_{q_\phi(\mathbf{u},\boldsymbol{\beta},\mathbf{z}|\mathbf{x})}[q_\phi(\mathbf{u}, \boldsymbol{\beta}, \mathbf{z}|\mathbf{x})].$$

Recall: Domain index definition (informal)

- Independence: independent of the data's encoding z
- Information Preservation: retain as much information on the data x as possible.
- Label sensitivity: maximize adaptation performance.

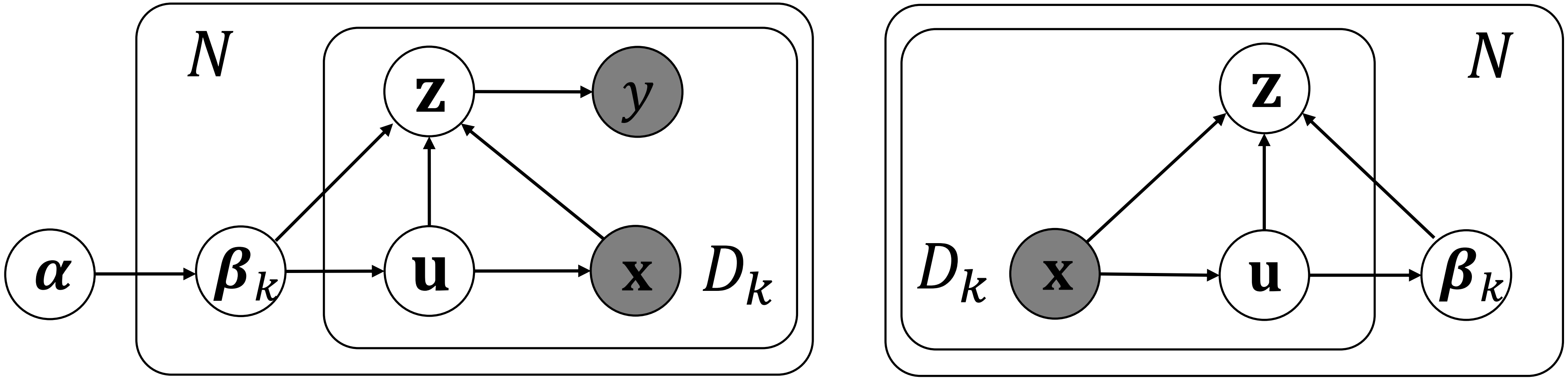
Adversarial loss



$$\mathcal{L}_{D,\phi} = \mathbb{E}_{p(k,\mathbf{x})}\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log D(k|\mathbf{z})]$$

Adversarial loss: enforce $\beta \perp\!\!\!\perp \mathbf{z}$ (Independence)

Final objective function



$$\max_{\theta,\phi}\min_{D}\ \mathcal{L}_{VDI} = \max_{\theta,\phi}\min_{D}\ \mathcal{L}_{\theta,\phi} - \lambda_d \mathcal{L}_{D,\phi}$$

$$= \max_{\theta,\phi}\min_{D}\ \underbrace{\mathbb{E}_{p(\mathbf{x},y)}[\mathcal{L}_{ELBO}(\mathbf{x},y)]}_{\text{ELBO}} - \lambda_d \underbrace{\mathbb{E}_{p(k,\mathbf{x})}\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log D(k|\mathbf{z})]}_{\text{Adversarial loss}}$$

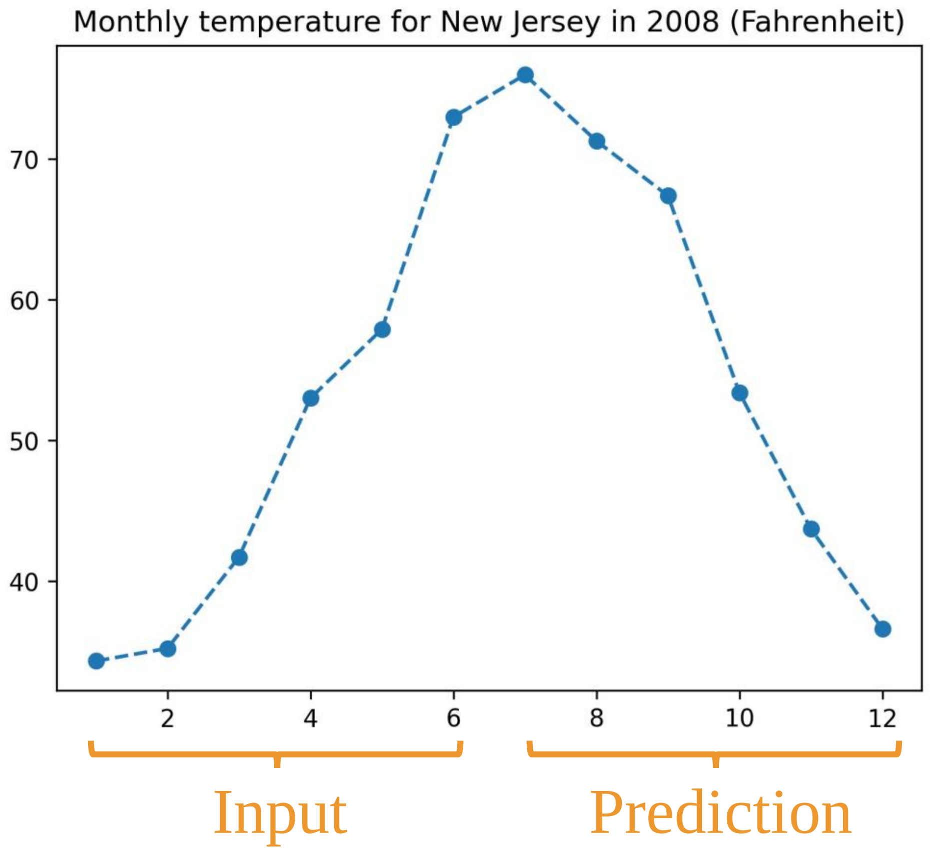
$$\max_{\theta,\phi}\min_{D}\ \mathcal{L}_{VDI} = \max_{\theta,\phi}\min_{D}\ \mathcal{L}_{\theta,\phi} - \lambda_d \mathcal{L}_{D,\phi}$$
$$= \max_{\theta,\phi}\min_{D}\ \mathbb{E}_{p(\mathbf{x},y)}[\mathcal{L}_{ELBO}(\mathbf{x},y)] - \lambda_d \mathbb{E}_{p(k,\mathbf{x})}\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x})}[\log D(k|\mathbf{z})]$$

Theorem (informal)

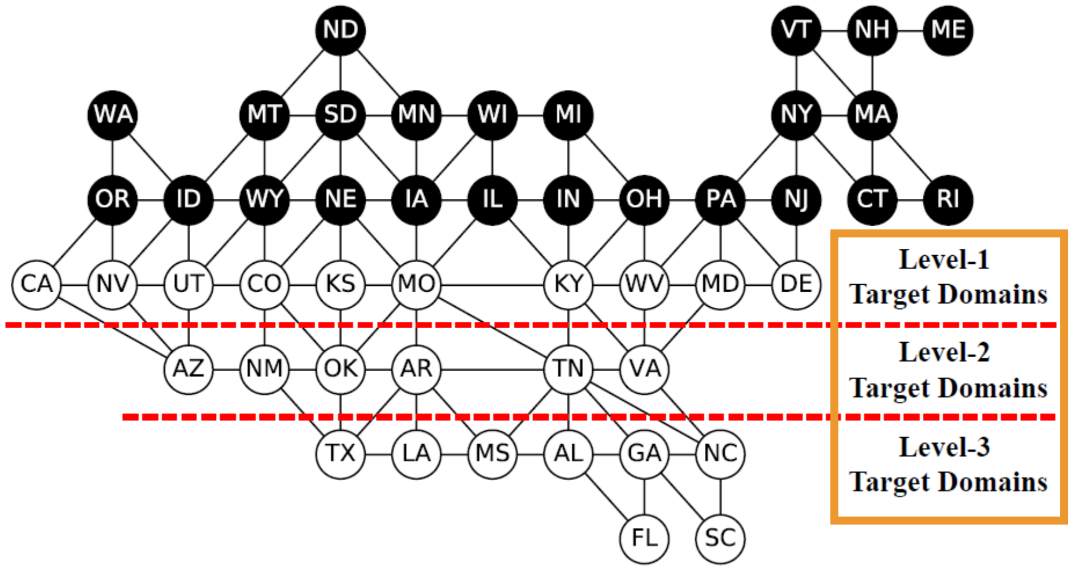
Using this objective function, we could learn domain indices β according to the previous definition.

- Independence
- Information Preservation
- Label sensitivity

TPT-48

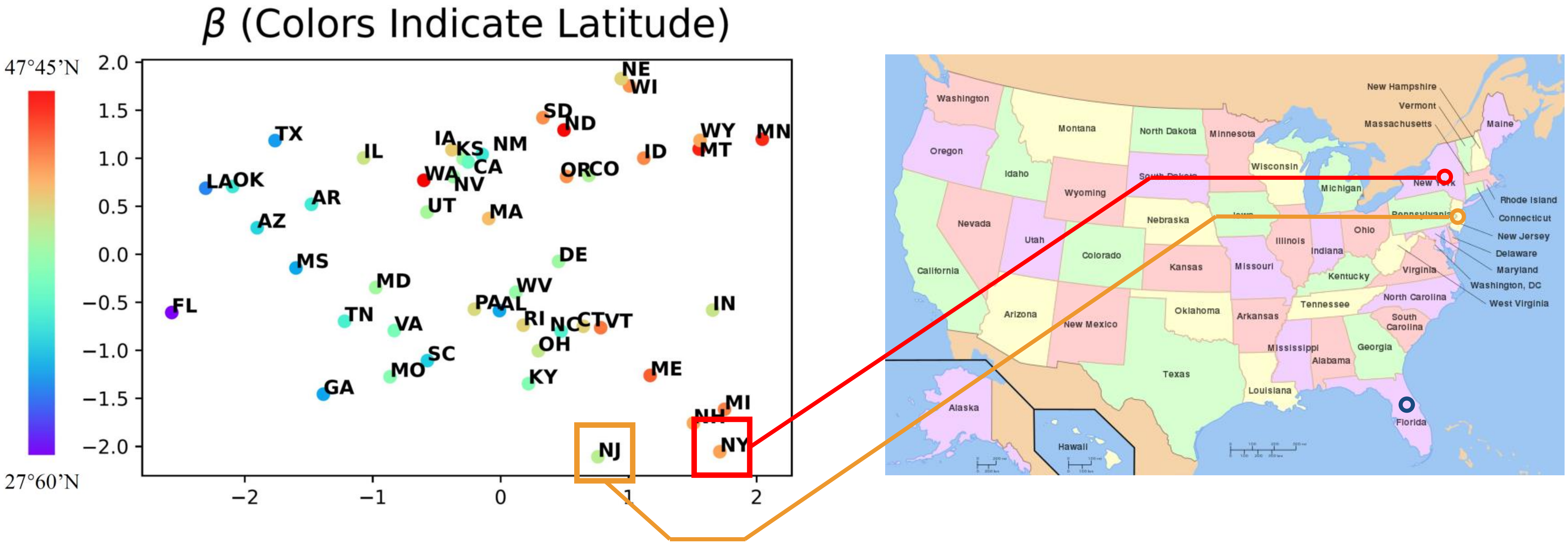


Task: temperature prediction



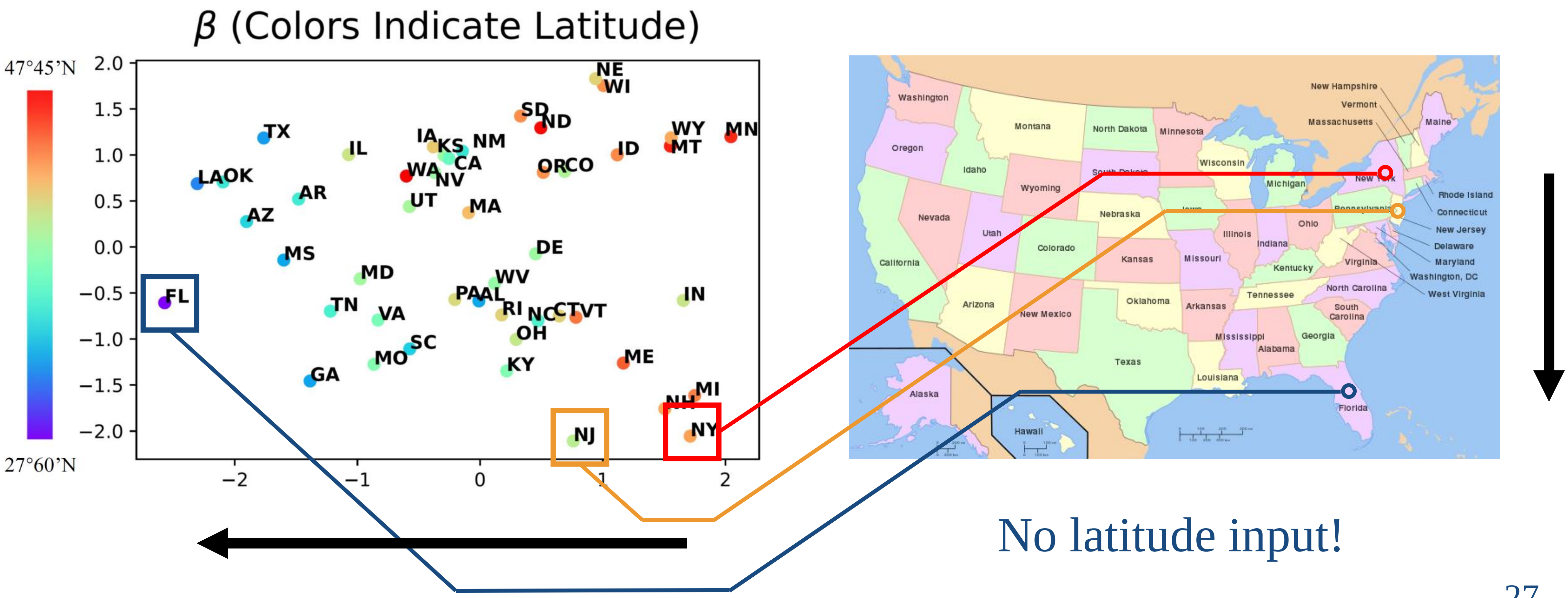
N (24) → S (24)

Domain index visualization



Experiment

Domain index visualization



No latitude input!

Performance of VDI

TPT-48:

Task	Domain	Source-Only	DANN	ADDA	CDANN	MDD	SENTRY	VDI (Ours)
N (24)→S (24)	Average of 10 Level-1 Domains	0.206	0.229	0.734	0.229	0.342	0.497	**0.192**
	Average of 6 Level-2 Domains	0.391	0.412	0.861	0.357	0.768	0.470	**0.323**
	Average of 8 Level-3 Domains	1.160	0.843	0.886	0.961	1.326	**0.459**	0.703
	Average of All 24 Domains	0.570	0.480	0.816	0.505	0.777	0.477	**0.395**

Conclusion

Take home message:

- VDI provides a principled way to infer the domain index.
- VDI improves both interpretability and performance of domain adaptation.
- VDI has theoretical guarantee.

Supplement

Code

https://github.com/Wang-ML-Lab/VDI

Paper

https://arxiv.org/abs/2302.02561

Thank you!

Q & A